2000 MATH OLYMPICS

LEVEL II

1. Olivia had a math quiz every school day. *In addition to studying*, she dropped some coins in a fountain on her way to school for good luck. However, she also felt that throwing in the same amount twice would bring bad luck. What is the maximum number of visits she could make to the fountain to toss in a total of 100 pennies so that the quantity of coins differed from the previous tosses?

   a. 11 b. 12 c. 13 d. 14 e. None of the above

2. Solve $x = \cfrac{2}{1 + \cfrac{2}{1 + \cfrac{2}{1 + \ddots}}}$ for x.

   a. 1 b. 2 c. 1 and 2 d. $\frac{1 \pm \sqrt{5}}{2}$ e. None of the above

3. Find all solutions of the equation $2 \cos t + 1 = 0$ in the interval $[0, 2\pi)$.

   a. $\frac{2\pi}{3}, \frac{4\pi}{3}$ b. $\frac{4\pi}{3}, \frac{5\pi}{3}$ c. $\frac{5\pi}{6}, \frac{7\pi}{6}$ d. $\frac{7\pi}{6}, \frac{11\pi}{6}$ e. None of the above

4. Trees A and B are 500 feet apart on the same side of a swamp. On the opposite side of the swamp is a tree C. If the angle between the lines AB and AC is 105° and that between AB and BC is 30°, find the distance between trees A and C.

   a. $\frac{500}{\sqrt{3}}$ *ft* b. $\frac{1000}{\sqrt{2}}$ *ft* c. $250\sqrt{3}$ *ft* d. $250\sqrt{2}$ *ft* e. None of the above

5. Find the term involving x^{12} in the expansion of $(x^2 + 2y)^9$.

   a. $1008x^{12}y^3$ b. $672x^{12}y^3$ c. $126x^{12}y^3$ d. $84x^{12}y^3$ e. None of the above

6. How many of the following statements are true?

   i) $\sin\alpha > \sec\alpha$ for some angle α
   ii) $\ln e \geq \log_3 4$
   iii) $\log(10x) = 1 + \log x$ for all x in the domain
   iv) $\tan(x + \pi) = \tan x + \tan\pi$ for all x

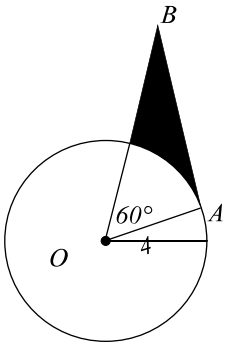
   a. 0 b. 1 c. 2 d. 3 e. 4

7. If $\frac{e^t + e^{-t}}{e^t - e^{-t}} = y$ then t is equal to

   a. $\ln(y+1) - \ln(y-1)$ b. $\frac{1}{2}[\ln(y+1) - \ln(y-1)]$ c. $-\frac{1}{2}\ln y$

   d. 1 e. None of the above

8. In the figure below, $\overline{BA}$ is tangent to circle O at A. What is the area of the shaded region?



   a. $\frac{8}{3}\pi$ b. $8 - \frac{1}{3}\pi$ c. $8\sqrt{3} - \frac{8}{3}\pi$

   d. $16 - \frac{1}{3}\pi$ e. None of the above

9. The sum of the real solutions to $\log(x^2 - 11) - \log x = 1$ is

   a. 1 b. 10 c. 11 d. No real solutions e. None of the above

10. The sum of the real roots, counting multiplicity, of $x^5 - 3x^3 + 2x^2 = 0$ is

    a. 0 b. 1 c. 2 d. 4 e. None of the above

11. Write $\csc(\cos^{-1} x)$ as an algebraic expression in x, free of trigonometric or inverse trigonometric functions.

    a. $\frac{1}{x}$ b. $\frac{1}{\sqrt{1-x^2}}$ c. $\frac{x}{\sqrt{1-x^2}}$ d. $\sqrt{1-x^2}$ e. None of the above

12. If $f(x) = 2^x$, then $f(1) + f(2) + f(3) + f(4) + \cdots + f(20)$ is equal to

a. $2^{21} - 2$ b. 2^{21} c. 40^{10} d. $40^{10} - 2$ e. None of the above

13. The number of intersection points of the graphs of the polar equations $r = 4\sin\theta$ and $r = 4\cos\theta$ is

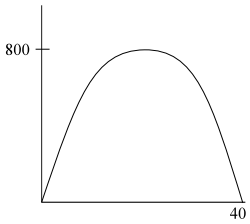
a. 0 b. 1 c. 2 d. 4 e. None of the above

14. Five distinct points are selected on the circumference of a circle. How many triangles can be drawn using these five points as vertices?

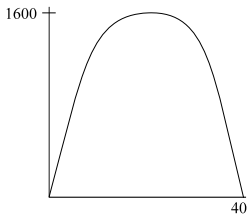
a. 5 b. 10 c. 15 d. 20 e. None of the above

15. A homeowner has 80 feet of chain-link fencing to be used to construct a dog pen adjacent to a house (see the figure). Which graph best describes the area, $A(x)$, of the enclosed area?

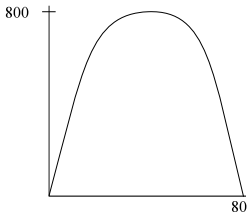
a.



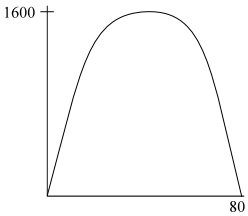
b.



c.



d.

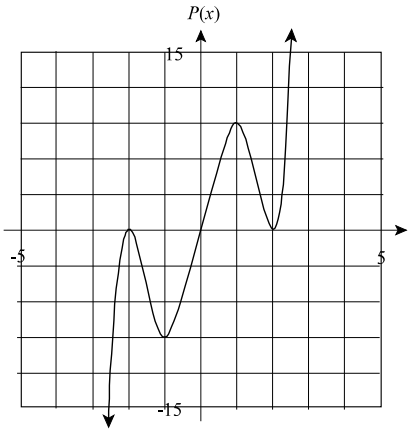


e. None of the above are possible

16. For $f(x) = \sqrt{x+4}$, the domain of $f^{-1}(x)$ is

a. $[-4,\infty)$ b. $(-4,\infty)$ c. $[0,\infty)$ d. $(-\infty,\infty)$ e. None of the above

17. What is the lowest possible degree of the polynomial whose graph is shown?



a. 3 b. 4 c. 5

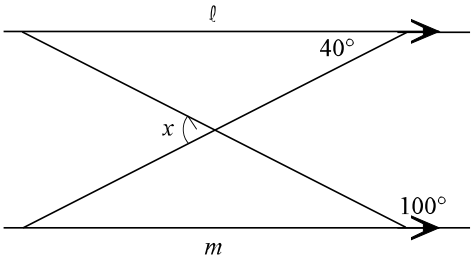
d. 6 e. None of the above

18. The fourth number in the list is missing. Find the sum of the digits of this missing number.

5 11 23 ??? 95 191

a. 5 b. 11 c. 12 d. 14 e. None of the above

19. If line ℓ is parallel to line m, how big is the angle marked x?



a. 40° b. 80° c. 100°

d. 120° e. None of the above

20. In a bag there are six white balls and four black balls. Two balls are taken out at random, one at a time, without replacement. What is the probability that the *second* ball taken out is black?

a. 2/5 b. 4/9 c. 2/3 d. 1/3 e. None of the above

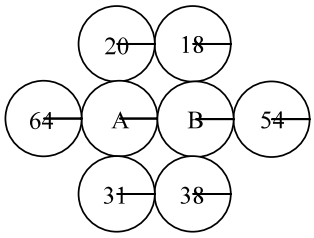
21. $\sin\left(\frac{\pi}{2} - x\right) =$

a. $\cos x$ b. $-\cos x$ c. $1 - \sin x$ d. $\sin x - 1$ e. None of the above

22. Ali (A) and Baba (B) are surrounded by six thieves (as shown). The thieves' ages are given. Ali's age is the average of his four nearest neighbors, and so is Baba's. How old is Baba?

a. 37 b. 37½ c. 38

d. 38⅓ e. None of the above



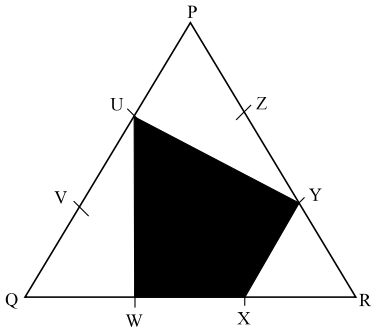
23. A car can go r miles on s gallons of gasoline. How many gallons of gas would it need for a t-mile trip?

a. $\frac{rs}{t}$ b. $\frac{st}{r}$ c. $\frac{rt}{s}$ d. $\frac{t}{rs}$ e. None of the above

24. PQR is an equilateral triangle. The points U, V, W, X, Y, Z trisect the sides. What is the ratio of the area of the shaded quadrilateral $UWXY$ to the area of the whole triangle PQR?

a. 1:2 b. 1:3 c. 2:5

d. 4:9 e. None of the above



25. Recall that 50! represents the product of all the whole numbers from 1 to 50 inclusive. If you were to calculate the actual value of 50!, how many zeros would the answer have at the end?

a. 5 b. 10 c. 11 d. 12 e. None of the above